

# An Overview of the Asymptotic Performance of the Family of the FastICA Algorithms

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**Abstract.** This contribution summarizes the results on the asymptotic performance of several variants of the FastICA algorithm. A number of new closed-form expressions are presented.

**Keywords:** Independent component analysis, symmetric FastICA, deflationary FastICA, data whitening, data centering, asymptotic performance.

## 1 Introduction

In what follows, we denote scalars by lowercase letters ( $a, b, c, \dots$ ), vectors by boldface lowercase letters ( $\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots$ ) and matrices by boldface uppercase letters ( $\mathbf{A}, \mathbf{B}, \mathbf{C}, \dots$ ). Greek letters ( $\alpha, \beta, \gamma, \dots$ ) are reserved for particular scalar quantities. We denote by  $\mathbf{A}^\top$  the matrix transpose of  $\mathbf{A}$  and by  $\|\cdot\|$  the Euclidean norm.

### 1.1 ICA Data Model

We consider the following noiseless linear ICA model:

$$\mathbf{y}(t) = \mathbf{H}\mathbf{s}(t), \quad t = 1, \dots, N,$$

where

1.  $\mathbf{s}(t) \stackrel{\text{def}}{=} (s_1(t), \dots, s_d(t))^\top$  denotes the  $t$ th realization of the unknown *source signal*. The components  $s_1(t), \dots, s_d(t)$  are mutually statistically independent, have unit variance and at most one of them is Gaussian. Furthermore,  $\mathbf{s}(1), \dots, \mathbf{s}(N)$  denote  $N$  independent realizations of  $\mathbf{s}$ .
2.  $\mathbf{y}(t) \stackrel{\text{def}}{=} (y_1(t), \dots, y_d(t))^\top$  denotes the  $t$ th realization of the *observed signal*.
3.  $\mathbf{H} \in \mathbb{R}^{d \times d}$  is a full rank square matrix, called the mixing matrix.

## 1.2 Data Preprocessing

Most ICA methods require the observed signal  $\{\mathbf{y}(t)\}$  to be standardized [1,2,3]. The standardization of  $\{\mathbf{y}(t)\}$  consists of the data centering and data whitening, which involve the estimation of  $\mathbb{E}[\mathbf{y}]$  and  $\text{Cov}(\mathbf{y})$ . In practice,  $\mathbb{E}[\mathbf{y}]$  and  $\text{Cov}(\mathbf{y})$  are usually estimated by the sample mean and sample variance:

$$\bar{\mathbf{y}} \stackrel{\text{def}}{=} \sum_{t=1}^N \frac{1}{N} \mathbf{y}(t), \quad \hat{\mathbf{C}} \stackrel{\text{def}}{=} \frac{1}{N} \sum_{t=1}^N (\mathbf{y}(t) - \bar{\mathbf{y}})(\mathbf{y}(t) - \bar{\mathbf{y}})^\top.$$

In this work, we shall consider several different data preprocessing scenarios. Denote

$$\tilde{\mathbf{C}} = \frac{1}{N} \sum_{t=1}^N (\mathbf{y}(t) - \mathbb{E}[\mathbf{y}])(\mathbf{y}(t) - \mathbb{E}[\mathbf{y}])^\top.$$

The following data preprocessing scenarios will be studied:

- 1). Theoretical whitening and theoretical centering.

$$\mathbf{x}(t) \stackrel{\text{def}}{=} \text{Cov}(\mathbf{y})^{-\frac{1}{2}} (\mathbf{y}(t) - \mathbb{E}[\mathbf{y}]). \quad (1)$$

- 2). Theoretical whitening and empirical centering.

$$\mathbf{x}(t) \stackrel{\text{def}}{=} \text{Cov}(\mathbf{y})^{-\frac{1}{2}} (\mathbf{y}(t) - \bar{\mathbf{y}}). \quad (2)$$

- 3). Empirical whitening and theoretical centering.

$$\mathbf{x}(t) \stackrel{\text{def}}{=} \tilde{\mathbf{C}}^{-\frac{1}{2}} (\mathbf{y}(t) - \mathbb{E}[\mathbf{y}]). \quad (3)$$

- 4). Empirical whitening and empirical centering.

$$\mathbf{x}(t) \stackrel{\text{def}}{=} \hat{\mathbf{C}}^{-\frac{1}{2}} (\mathbf{y}(t) - \bar{\mathbf{y}}). \quad (4)$$

In the sequel,  $\mathbf{x}(t)$  will always stand for the standardized signal under one of the scenarios defined above. The specific data preprocessing scenario will be stated explicitly when necessary.

## 1.3 Variants of the FastICA Algorithm

Before proceeding further, we need to introduce some notations first. We denote by  $\mathcal{S}$  the unit sphere in  $\mathbb{R}^d$ . We denote by  $g(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$  the nonlinearity function, and by  $G(\cdot)$  its primitive. The nonlinearity function  $g$  is usually supposed to be non-linear, non-quadratic and smooth. For any function  $f : \mathbb{R}^d \rightarrow \mathbb{R}^m$ , we write  $\hat{\mathbb{E}}_{\mathbf{x}}[f(\mathbf{x})] \stackrel{\text{def}}{=} \frac{1}{N} \sum_{t=1}^N f(\mathbf{x}(t))$  for conciseness.

**The Deflationary FastICA Algorithm** This version of the FastICA algorithm extracts the sources sequentially. It consists of the following steps [3]:

- **Input:**  $\mathbf{x}(1), \dots, \mathbf{x}(N)$ .
- 1). Set  $p = 1$ .
- 2). Choose an arbitrary initial iterate  $\mathbf{w} \in \mathcal{S}$ ;
- 3). Run iteration

$$\mathbf{w} \leftarrow \hat{\mathbb{E}}_{\mathbf{x}}[g'(\mathbf{w}^T \mathbf{x})\mathbf{w} - g(\mathbf{w}^T \mathbf{x})\mathbf{x}] \quad (5)$$

$$\mathbf{w} \leftarrow \mathbf{w} - \sum_{i=1}^{p-1} (\mathbf{w}_i^{DFL})^T \mathbf{w} \quad (6)$$

$$\mathbf{w} \leftarrow \frac{\mathbf{w}}{\|\mathbf{w}\|} \quad (7)$$

until convergence<sup>1</sup>. The limit is stored as  $\mathbf{w}_p^{DFL}$ .

- 4). Break if  $p = d$ . Otherwise  $p \leftarrow p + 1$  then go to step 2).
- **Output:**  $\mathbf{W}^{DFL} = (\mathbf{w}_1^{DFL}, \dots, \mathbf{w}_d^{DFL})$ .

**The Symmetric FastICA Algorithm** The symmetric version of FastICA extracts all the sources simultaneously. It can be described as follows:

- **Input:**  $\mathbf{x}(1), \dots, \mathbf{x}(N)$ .
- 1). Choose an arbitrary orthonormal matrix  $\mathbf{W} = (\mathbf{w}_1, \dots, \mathbf{w}_d) \in \mathbb{R}^{d \times d}$ .
- 2). Run

$$\mathbf{w}_1 \leftarrow \hat{\mathbb{E}}_{\mathbf{x}}[g'(\mathbf{w}_1^T \mathbf{x})\mathbf{w}_1 - g(\mathbf{w}_1^T \mathbf{x})\mathbf{x}] \quad (8)$$

$\vdots$

$$\mathbf{w}_d \leftarrow \hat{\mathbb{E}}_{\mathbf{x}}[g'(\mathbf{w}_d^T \mathbf{x})\mathbf{w}_d - g(\mathbf{w}_d^T \mathbf{x})\mathbf{x}] \quad (9)$$

$$\mathbf{W} \leftarrow \left( \mathbf{W}\mathbf{W}^T \right)^{-1/2} \mathbf{W} \quad (10)$$

until convergence. The limit is denoted by  $\mathbf{W}^{SYM}$ .

- **Output:**  $\mathbf{W}^{SYM} = (\mathbf{w}_1^{SYM}, \dots, \mathbf{w}_d^{SYM})$ .

## 2 Asymptotic Performance

Let us introduce the notion of gain matrix:

$$\mathbf{G}^{DFL} \stackrel{\text{def}}{=} (\mathbf{W}^{DFL})^T \mathbf{C}^{-1/2} \mathbf{H}, \quad \mathbf{G}^{SYM} \stackrel{\text{def}}{=} (\mathbf{W}^{SYM})^T \mathbf{C}^{-1/2} \mathbf{H},$$

where  $\mathbf{C}^{-1/2}$  stands for the sphering matrix used in the data preprocessing stage, i.e.  $\mathbf{C} = \text{Cov}(\mathbf{y})$  in scenarios (1) and (2),  $\mathbf{C} = \hat{\mathbf{C}}$  in scenario (3) and  $\mathbf{C} = \hat{\mathbf{C}}$  in scenario (4). Without loss of generality, we shall omit the permutation and

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<sup>1</sup> We impose the number of iterations to be even, so that the well known sign-flipping phenomenon disappears.

sign ambiguities of ICA. Then,  $\mathbf{G}^{DFL} \approx \mathbf{I}$  and  $\mathbf{G}^{SYM} \approx \mathbf{I}$ , hence  $\mathbf{C}^{-1/2}\mathbf{W}^{DFL}$  and  $\mathbf{C}^{-1/2}\mathbf{W}^{SYM}$  can be considered as estimators of  $\mathbf{B} \stackrel{\text{def}}{=} (\mathbf{H}^{-1})^\top$ . In the sequel, we will study the asymptotic errors of  $N^{1/2}(\mathbf{C}^{-1/2}\mathbf{W}^{DFL} - \mathbf{B})$  and  $N^{1/2}(\mathbf{C}^{-1/2}\mathbf{W}^{SYM} - \mathbf{B})$  under proposed data preprocessing scenarios.

The proofs of the results presented below are based on the method of M-estimators. However, all proofs will be omitted due to the lack of space. A complete version of this work can be provided upon request. The readers are also referred to [4] for a more detailed account of this subject.

## 2.1 The Asymptotic Error of Deflationary FastICA

Assume that the following mathematical expectations exist for  $i = 1, \dots, d$ :

$$\begin{aligned}\alpha_i &\stackrel{\text{def}}{=} \mathbb{E}[g'(z_i) - g(z_i)z_i] \\ \beta_i &\stackrel{\text{def}}{=} \mathbb{E}[g(z_i)^2] \\ \gamma_i &\stackrel{\text{def}}{=} \mathbb{E}[g(z_i)z_i] \\ \eta_i &\stackrel{\text{def}}{=} \mathbb{E}[g(z_i)] \\ \tau_i &\stackrel{\text{def}}{=} (\mathbb{E}[z_i^4] - 1)/4,\end{aligned}$$

where  $z_i = s_i - \mathbb{E}[s_i]$  for  $i = 1, \dots, d$ .

**Theorem 1.** *Let  $\mathbf{b}_i$  denote the  $i$ th column of  $\mathbf{B}$ . Under some mild regularity conditions, we have*

$$N^{1/2}(\mathbf{C}^{-1/2}\mathbf{w}_i^{DFL} - \mathbf{b}_i) \xrightarrow[N \rightarrow \infty]{\mathcal{D}} \mathcal{N}(0, \mathbf{R}_{(k)}^{DFL}),$$

where  $k \in \{1, 2, 3, 4\}$  is the label of the underlying data preprocessing scenario (see (1)-(4)) and  $\mathbf{R}_{(k)}^{DFL}$  is given as follows:

$$\mathbf{R}_{(1)}^{DFL} = \sum_{j=1}^{i-1} \frac{\beta_j^2}{\alpha_j^2} \mathbf{b}_j \mathbf{b}_j^\top + \sum_{\substack{p,q=1 \\ p \neq q}}^{i-1} \frac{\eta_p \eta_q}{\alpha_p \alpha_q} \mathbf{b}_p \mathbf{b}_q^\top + \frac{\beta_i^2}{\alpha_i^2} \sum_{j=i+1}^d \mathbf{b}_j \mathbf{b}_j^\top, \quad (11)$$

$$\mathbf{R}_{(2)}^{DFL} = \sum_{j=1}^{i-1} \frac{\beta_j - \eta_j^2}{\alpha_j^2} \mathbf{b}_j \mathbf{b}_j^\top + \frac{\beta_i - \eta_i^2}{\alpha_i^2} \sum_{j=i+1}^d \mathbf{b}_j \mathbf{b}_j^\top, \quad (12)$$

$$\begin{aligned}\mathbf{R}_{(3)}^{DFL} &= \sum_{j=1}^{i-1} \frac{\beta_j - \gamma_j^2 + \alpha_j^2}{\alpha_j^2} \mathbf{b}_j \mathbf{b}_j^\top + \sum_{\substack{p,q=1 \\ p \neq q}}^{i-1} \frac{\eta_p \eta_q}{\alpha_p \alpha_q} \mathbf{b}_p \mathbf{b}_q^\top + \tau_i \mathbf{b}_i \mathbf{b}_i^\top \\ &\quad + \frac{\beta_i - \gamma_i^2}{\alpha_i^2} \sum_{j=i+1}^d \mathbf{b}_j \mathbf{b}_j^\top - \sum_{j=1}^{i-1} \frac{\mathbb{E}[s_i^3] \eta_j}{\alpha_j} (\mathbf{b}_j \mathbf{b}_i^\top + \mathbf{b}_i \mathbf{b}_j^\top),\end{aligned} \quad (13)$$

$$\begin{aligned}\mathbf{R}_{(4)}^{DFL} &= \sum_{j=1}^{i-1} \frac{\beta_j - \gamma_j^2 + \alpha_j^2 - \eta_j^2}{\alpha_j^2} \mathbf{b}_j \mathbf{b}_j^\top + \tau_i \mathbf{b}_i \mathbf{b}_i^\top + \frac{\beta_i - \gamma_i^2 - \eta_i^2}{\alpha_i^2} \sum_{j=i+1}^d \mathbf{b}_j \mathbf{b}_j^\top \\ &\quad - \sum_{j=1}^{i-1} \frac{\mathbb{E}[s_i^3] \eta_j}{\alpha_j} (\mathbf{b}_j \mathbf{b}_i^\top + \mathbf{b}_i \mathbf{b}_j^\top).\end{aligned} \quad (14)$$

**Corollary 2.** *There holds  $N^{1/2}(\mathbf{G}_{ij}^{DFL} - \delta_{ij}) \xrightarrow[N \rightarrow \infty]{\mathcal{D}} \mathcal{N}(0, V_{(k)}^{DFL})$ , where  $\mathbf{G}_{ij}^{DFL}$  denotes the  $(i, j)$ th entry of  $\mathbf{G}^{DFL}$  and  $V_{(k)}^{DFL}$  is given as follows:*

1. *Case  $j < i$ :*

$$\begin{aligned} V_{(1)}^{DFL} &= \frac{\beta_j^2}{\alpha_j^2} \\ V_{(2)}^{DFL} &= \frac{\beta_j - \eta_j^2}{\alpha_j^2} \\ V_{(3)}^{DFL} &= \frac{\beta_j - \gamma_j^2 + \alpha_j^2}{\alpha_j^2} \\ V_{(4)}^{DFL} &= \frac{\beta_j - \gamma_j^2 + \alpha_j^2 - \eta_j^2}{\alpha_j^2}. \end{aligned}$$

2. *Case  $j = i$ :*

$$V_{(1)}^{DFL} = V_{(2)}^{DFL} = 0, \quad V_{(3)}^{DFL} = V_{(4)}^{DFL} = \tau_i.$$

3. *Case  $j > i$ :*

$$V_{(1)}^{DFL} = \frac{\beta_i}{\alpha_i^2} \tag{15}$$

$$V_{(2)}^{DFL} = \frac{\beta_i - \eta_i^2}{\alpha_i^2} \tag{16}$$

$$V_{(3)}^{DFL} = \frac{\beta_i - \gamma_i^2}{\alpha_i^2} \tag{17}$$

$$V_{(4)}^{DFL} = \frac{\beta_i - \gamma_i^2 - \eta_i^2}{\alpha_i^2}. \tag{18}$$

## 2.2 The Asymptotic Error of Symmetric FastICA

**Theorem 3.** *Under some mild regularity conditions, we have  $N^{1/2}(\mathbf{C}^{-1/2} \mathbf{w}_i^{SYM} - \mathbf{b}_i) \xrightarrow[N \rightarrow \infty]{\mathcal{D}} \mathcal{N}(0, \mathbf{R}_{(k)}^{SYM})$ , where*

$$\mathbf{R}_{(1)}^{SYM} = \sum_{j \neq i}^d \frac{\beta_i + \beta_j - 2\gamma_i \gamma_j - 2\eta_j^2}{(|\alpha_i| + |\alpha_j|)^2} \mathbf{b}_j \mathbf{b}_j^\top + 2 \sum_{j \neq i}^d \frac{\eta_j \mathbf{b}_j}{|\alpha_i| + |\alpha_j|} \sum_{j \neq i}^d \frac{\eta_j \mathbf{b}_j^\top}{|\alpha_i| + |\alpha_j|}, \tag{19}$$

$$\mathbf{R}_{(2)}^{SYM} = \sum_{j \neq i}^d \frac{\beta_i + \beta_j - 2\gamma_i \gamma_j - 2\eta_i^2}{(|\alpha_i| + |\alpha_j|)^2} \mathbf{b}_j \mathbf{b}_j^\top, \tag{20}$$

$$\begin{aligned} \mathbf{R}_{(3)}^{SYM} &= \sum_{j \neq i}^d \frac{\beta_i - \gamma_i^2 + \beta_j - \gamma_j^2 + \alpha_j^2 - \eta_j^2}{(|\alpha_i| + |\alpha_j|)^2} \mathbf{b}_j \mathbf{b}_j^\top + \sum_{j \neq i}^d \frac{\eta_j \mathbf{b}_j}{(|\alpha_i| + |\alpha_j|)} \sum_{j \neq i}^d \frac{\eta_j \mathbf{b}_j^\top}{(|\alpha_i| + |\alpha_j|)} \\ &\quad + \tau_i \mathbf{b}_i \mathbf{b}_i^\top - \sum_{j \neq i}^d \frac{\mathbb{E}[s_i^3] \eta_j}{2(|\alpha_i| + |\alpha_j|)} (\mathbf{b}_j \mathbf{b}_i^\top + \mathbf{b}_i \mathbf{b}_j^\top), \end{aligned} \tag{21}$$

$$\mathbf{R}_{(4)}^{SYM} = \sum_{j \neq i}^d \frac{\beta_i - \gamma_i^2 + \beta_j - \gamma_j^2 + \alpha_j^2 - \eta_i^2 - \eta_j^2}{(|\alpha_i| + |\alpha_j|)^2} \mathbf{b}_j \mathbf{b}_j^\top + \tau_i \mathbf{b}_i \mathbf{b}_i^\top. \quad (22)$$

**Corollary 4.** For  $i, j = 1, \dots, d$ , there holds  $N^{1/2}(\mathbf{G}_{ij}^{SYM} - \delta_{ij}) \xrightarrow[N \rightarrow \infty]{\mathcal{D}} \mathcal{N}(0, V_{(k)}^{SYM})$ , where

1. Case  $j = i$ :

$$V_{(1)}^{SYM} = V_{(2)}^{SYM} = 0, \quad V_{(3)}^{SYM} = V_{(4)}^{SYM} = \tau_i.$$

2. Case  $j \neq i$ :

$$V_{(1)}^{SYM} = \frac{\beta_i + \beta_j - 2\gamma_i \gamma_j}{(|\alpha_i| + |\alpha_j|)^2}, \quad (23)$$

$$V_{(2)}^{SYM} = \frac{\beta_i + \beta_j - 2\gamma_i \gamma_j - 2\eta_i^2}{(|\alpha_i| + |\alpha_j|)^2}, \quad (24)$$

$$V_{(3)}^{SYM} = \frac{\beta_i - \gamma_i^2 + \beta_j - \gamma_j^2 + \alpha_j^2}{(|\alpha_i| + |\alpha_j|)^2}, \quad (25)$$

$$V_{(4)}^{SYM} = \frac{\beta_i - \gamma_i^2 + \beta_j - \gamma_j^2 + \alpha_j^2 - \eta_i^2 - \eta_j^2}{(|\alpha_i| + |\alpha_j|)^2}. \quad (26)$$

*Remark 5.* Although the asymptotic error of the FastICA algorithm has already been studied by quite a few researchers [5,6,7,8], many of the results presented in this contribution, notably expressions (11)-(13) established in Theorem 1 and (19)-(22) in Theorem 3, are new.

*Example 1.* The validity of formulas (15)-(18) and (23)-(26) is verified in computer simulations, see Fig. 1 and Fig. 2. The simulations are configured as follows:  $d = 3$ ,  $N = 5000$ , all three sources have identical bimodal Gaussian distribution with asymmetrical density. Both deflationary FastICA and symmetric FastICA have been tested with different data preprocessing (1)-(4) in 5000 independent trials.

### 2.3 Discussion

First, comparing the expressions in Corollary 2 and Corollary 4, we find that for the  $(i, j)$ th entry of the gain matrix,

$$V_{(1)}^{DFL} - V_{(2)}^{DFL} = V_{(3)}^{DFL} - V_{(4)}^{DFL} = \frac{\eta_j^2}{\alpha_j^2}, \quad j < i,$$

$$V_{(1)}^{DFL} - V_{(2)}^{DFL} = V_{(3)}^{DFL} - V_{(4)}^{DFL} = \frac{\eta_i^2}{\alpha_i^2}, \quad j > i,$$

$$V_{(1)}^{SYM} - V_{(2)}^{SYM} = \frac{2\eta_i^2}{(|\alpha_i| + |\alpha_j|)^2}, \quad i \neq j,$$

$$V_{(3)}^{SYM} - V_{(4)}^{SYM} = \frac{\eta_i^2 + \eta_j^2}{(|\alpha_i| + |\alpha_j|)^2}, \quad i \neq j.$$

Since all the differences above are non-negative<sup>2</sup>, we assert that the empirical data centering generally leads to a better asymptotic performance.

### 3 Conclusion

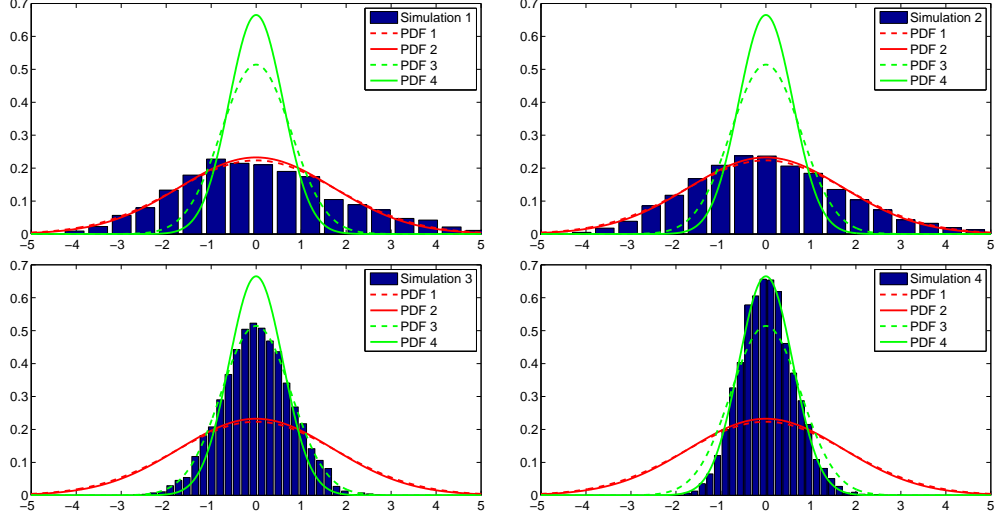
The contribution of this work is twofold. First, we derived explicit formulas for the asymptotic error of the two most important variants of the FastICA algorithm, the deflationary FastICA and the symmetric FastICA, under four different data preprocessing scenarios. Many of the presented formulas are novel. Second, we assessed the impact of empirical data preprocessing procedure on the asymptotic performance of the algorithms. We showed that, compared to the theoretical data centering, the empirical data centering generally leads to a better separation performance.

### References

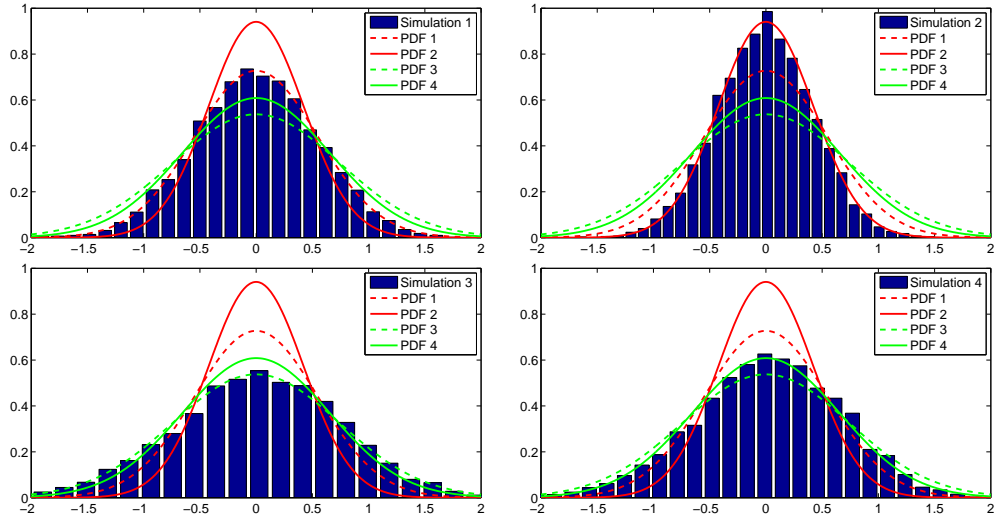
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<sup>2</sup> They become zero if  $\eta_i$  and/or  $\eta_j$  vanish. This is the case if, e.g.  $g$  is pair and the involved sources have symmetric distributions.



**Fig. 1.** Asymptotic error of the deflationary FastICA in each preprocessing scenario. We plotted the histograms of an (upper) off-diagonal entry of  $N^{1/2}\mathbf{G}^{DFL}$  in 5000 independent trials versus the theoretical curves of the Gaussian PDFs with variances given by (15)-(18).



**Fig. 2.** Asymptotic error of the symmetric FastICA in each preprocessing scenario. We plotted the histograms of an off-diagonal entry of  $N^{1/2}\mathbf{G}^{SYM}$  in 5000 independent trials versus the theoretical curves of the Gaussian PDFs with variances given by (23)-(26).